

6.5 Loans

A loan is an **ordinary** annuity from the bank's perspective; the bank invests a large **present value** in your home or car or education, and then withdraws payments, with interest, from you. We know this formula!

$$P = R \frac{1 - (1 + i)^{-n}}{i}$$

P = starting loan balance

R = loan payment

r = yearly rate,

m = compoundings/payments per year

$i = r/m$ = rate at each compounding

$n = mt$ = total payments

The process of repaying the principal and interest in equal payments is called amortization.

Example:

Sarah purchases a \$350,000 home with a down payment of \$75,000 and a mortgage loan for the rest at 5%, compounded monthly amortized over 30 years.

- (a) What is the starting loan balance?
- (b) What are the monthly payments?
- (c) What is the total amount Sarah pays for the house? (*i.e.* total over the “life of the loan”)
- (d) How much interest does she pay?

Don't need to write this down

Amortization (Payment) Schedule

for Sarah's example:

$$i = 0.05/12 = 0.004166\dots$$

For each monthly statement, the bank computes interest by multiplying i times the unpaid balance. That part of your payments is interest, the rest is subtracted from the principal

Month	Payment	Interest	Principal	Unpaid Balance
0				\$275,000.00
1	\$1,476.26	\$1,145.83	\$330.43	\$274,669.57
2	\$1,476.26	\$1,144.46	\$331.80	\$274,337.77
3	\$1,476.26	\$1,143.07	\$333.19	\$274,004.58
4	\$1,476.26	\$1,141.69	\$334.57	\$273,670.01
5	\$1,476.26	\$1,140.29	\$335.97	\$273,334.04
6	\$1,476.26	\$1,138.89	\$337.37	\$272,996.67
7	\$1,476.26	\$1,137.49	\$338.77	\$272,657.90

Chapter 6 Summary

1. Are there regular payments?
 - (a) NO: it's a LUMP SUM problem.
 - (b) YES: it's an ANNUITY problem.
- 2(a) For LUMP SUM, does it say:
 - (i) "simple interest"?
 - (ii) "compound continuously"?
 - (iii) "compounded m times a year"?
- 2(b) For ANNUITIES,
 - (i) payments at BEGINNING or END?
 - (ii) balance GROWING or SHRINKING?

Special notes on Annuities:

1. First compute
$$i = r/m = \text{rate used at each period.}$$
$$n = mt = \text{total number of payments}$$
2. Note:
$$R \cdot n = \text{total amount paid.}$$
For FV questions
$$\text{Total interest} = F - R \cdot n$$
For PV questions
$$\text{Total interest} = R \cdot n - P$$
3. Loans are present value, ordinary annuities. If there is a down payment on a loan, then
$$P = \text{original value} - \text{down payment}$$